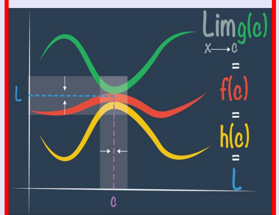
Calculus I

Lecture 25

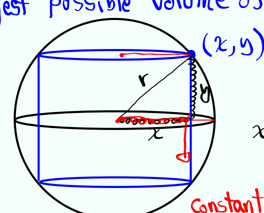


Feb 19-8:47 AM

Class Quiz 22

A right Circular cylinder is inscribed of a sphere of radius r.

Find the largest possible Volume of Such cylinder.



Volume of cylinder $\pi r^2 h$

$x^2+y^2=r^2$

$x^2=r^2-y^2$

$V=\pi(x)^2\cdot 2y$

$V=2\pi x^2 y$

Constant

$f(y)=2\pi(r^2-y^2)y$ $\qquad f(y)=2\pi\left[r^2y-y^3\right]$

$f'(y)=0$

$2\pi(r^2-3y^2)=0$

$y^2=\frac{r^2}{3} \quad y=\frac{r}{\sqrt{3}}$

$f'(y)=2\pi\left[r^2-3y^2\right]$

$f''(y)=2\pi\cdot -6y$

Since $y>0$, $f''(y)<0$

Max

$f(y)$ is Concave down

$x^2=r^2-y^2=r^2-\frac{r^2}{3}=\frac{2r^2}{3}$

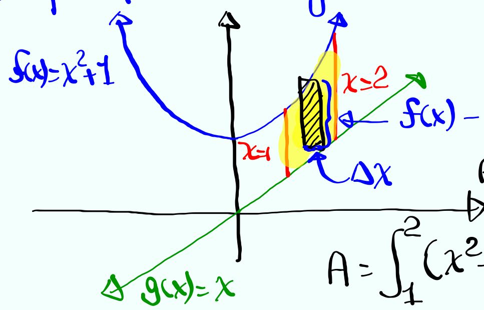
$V=\pi x^2\cdot 2y$

$=2\pi\cdot\frac{2r^2}{3}\cdot\frac{r}{\sqrt{3}}=\boxed{\frac{4\pi r^3}{3\sqrt{3}}}=\boxed{\frac{4\pi\sqrt{3}\,r^3}{9}}$

May 14-10:01 AM

Given $f(x) = x^2 + 1$ & $g(x) = x$

1) Graph & shade region between $x = 1$ & $x = 2$.



$f(x) - g(x) = x^2 + 1 - x$

$A = (x^2 + 1 - x) \cdot \Delta x$

$A = \int_1^2 (x^2 + 1 - x)\,dx$

$= \left(\frac{x^3}{3} + x - \frac{x^2}{2}\right)\Big|_1^2 = \left(\frac{2^3}{3} + 2 - \frac{2^2}{2}\right) - \left(\frac{1^3}{3} + 1 - \frac{1^2}{2}\right)$

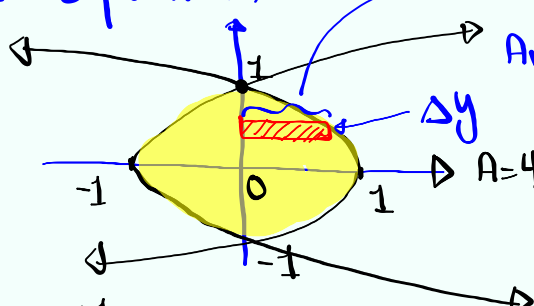
$= \boxed{\frac{8}{3}} + 2 - \frac{4}{2} \boxed{- \frac{1}{3}} - 1 + \frac{1}{2}$

$= \frac{7}{3} + 1 - \frac{3}{2} = \frac{14 + 6 - 9}{6} = \boxed{\frac{11}{6}}$

May 19-9:02 AM

Given $x = 1 - y^2$ & $x = y^2 - 1$

1) Graph & shade the region bounded by these 2 equations.



Right − left $= 1 - y^2 - 0$

Area $= (1 - y^2) \cdot \Delta y$

$A = 4\int_0^1 (1 - y^2)\,dy$

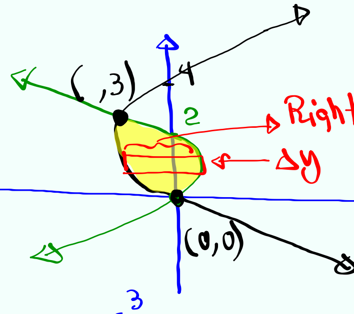
$A = 4\left[y - \frac{y^3}{3}\right]\Big|_0^1 = 4\left(1 - \frac{1}{3} - 0\right) = 4 \cdot \frac{2}{3} = \boxed{\frac{8}{3}}$

May 19-9:10 AM

Find the region bounded by $x = y^2 - 4y$ and $x = 2y - y^2$.

Parabola open left

Parabola opens right



Right − left $= 2y - y^2 - y^2 + 4y$

$= 6y - 2y^2$

Δy

$y^2 - 4y = 2y - y^2$

$2y^2 - 6y = 0$

$y^2 - 3y = 0$

$y = 0,\ y = 3$

$$A = \int_0^3 (6y - 2y^2)\,dy$$

$$= \left(\frac{6y^2}{2} - \frac{2y^3}{3}\right)\Big|_0^3$$

$$= 3(3)^2 - \frac{2(3)^3}{3} - 0$$

$$= 27 - 18 = \boxed{9}$$

May 19-9:19 AM

Integration by Substitution

$$\int (x+4)\,dx = \frac{x^2}{2} + 4x + C$$

$u = x + 4$ $\qquad$ $du = dx$

$u = x + 4$

$\frac{du}{dx} = 1 + 0$

$\frac{dy}{dx} = 1$

$du = dx$

$$\int u\,du = \frac{u^2}{2} + C = \frac{(x+4)^2}{2} + C$$

$$= \frac{x^2 + 8x + 16}{2} + C$$

$$= \frac{x^2}{2} + \frac{8x}{2} + \frac{16}{2} + C$$

$$= \frac{x^2}{2} + 4x + C$$

May 19-9:29 AM

$$\int 3(x+4)^2\,dx$$

$u = x+4 \qquad du = dx$

$$= \int 3u^2\,du = u^3 + C$$
$$= \boxed{(x+4)^3 + C}$$

$$\int \sqrt{1+2x}\,dx$$

$u = 1+2x \qquad du = 2dx \qquad \frac{du}{2} = dx$

$$= \int \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$$
$$= \frac{1}{2}\cdot\frac{2}{3}\,u^{3/2} + C$$
$$= \boxed{\frac{1}{3}(1+2x)^{3/2} + C}$$
$$= \boxed{\frac{1}{3}(1+2x)\sqrt{1+2x} + C}$$

May 19-9:34 AM

$$\int x\cos x^2\,dx$$

$u = x^2 \qquad du = 2x\,dx \qquad \frac{du}{2} = x\,dx$

$$\int \cos u \cdot \frac{du}{2} = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C$$

multiplication

$$= \boxed{\frac{1}{2}\sin x^2 + C}$$

May 19-9:40 AM

$$\int x^2\left(1+\tfrac{1}{3}x^3\right)^5 dx$$

$u = 1 + \frac{1}{3}x^3$

$du = \frac{1}{3}\cdot 3x^2 dx$

$du = x^2 dx$

$$= \int u^5\, du = \frac{u^6}{6} + C$$

$$= \frac{1}{6}\left(1+\tfrac{1}{3}x^3\right)^6 + C$$

$$\frac{d}{dx}\left[\frac{1}{6}\left(1+\tfrac{1}{3}x^3\right)^6 + C\right] = \cancel{\frac{1}{6}}\cdot\cancel{6}\left(1+\tfrac{1}{3}x^3\right)^5\cdot\left(0+\cancel{\tfrac{1}{3}}\cdot\cancel{3}x^2\right)+0$$

$$= \boxed{x^2\left(1+\tfrac{1}{3}x^3\right)^5}$$

May 19-9:45 AM

$$\int x^3\left(1+x^4\right)^9 dx$$

$u = 1 + x^4$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$$= \int u^9 \frac{du}{4}$$

$$= \frac{1}{4}\,\frac{u^{10}}{10} + C = \boxed{\frac{1}{40}\left(1+x^4\right)^{10} + C}$$

May 19-9:50 AM

$$\int \cos^3 x \underbrace{\sin x\, dx}$$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$$= \int u^3 (-du)$$

$$= -\int u^3 du = -\frac{u^4}{4} + C = \boxed{\frac{-1}{4}\cos^4 x + C}$$

$$\int \cos^3 x \sin x\, dx = \int \cos^2 x \cdot \cos x \cdot \sin x\, dx$$

$$= \int (1 - \sin^2 x) \sin x \cdot \cos x\, dx$$

$u = \sin x$

$du = \cos x\, dx$

$$= \int (1 - u^2) u\, du = \int (u - u^3) du$$

$$= \frac{u^2}{2} - \frac{u^4}{4} + C$$

$$= \boxed{\frac{1}{2}\sin^2 x - \frac{1}{4}\sin^4 x + C}$$

May 19-9:57 AM

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

$u = \sqrt{x}$

$u^2 = x \qquad 2u\,du = dx$

$$= \int \frac{\sin u}{\cancel{u}}\, 2\cancel{u}\, du = 2\int \sin u\, du$$

$$= 2(-\cos u) + C$$

$$= \boxed{-2\cos\sqrt{x} + C}$$

May 19-10:05 AM

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\,dx \qquad u = \frac{1}{x} \qquad du = \frac{-1}{x^2}\,dx$$

$$-du = \frac{1}{x^2}\,dx$$

$$= \int \cos u\,(-du) = -\int \cos u\,du$$

$$= -\sin u + C$$

$$= \boxed{-\sin\frac{1}{x} + C}$$

May 19-10:12 AM

$$\int \cos x\,\sec^2(\sin x)\,dx \qquad u = \sin x$$

$$du = \cos x\,dx$$

$$= \int \sec^2 u\,du = \tan u + C$$

$$= \tan(\sin x) + C$$

May 19-10:17 AM

$$\int \sqrt{\cot x}\ \csc^2 x\, dx$$

$u = \cot x$

$du = -\csc^2 x\, dx$

$-du = \csc^2 x\, dx$

$$= \int \sqrt{u}\,(-du) = -\int u^{1/2}\, du$$

$$= -\frac{u^{3/2}}{3/2} + C = \frac{-2}{3} u^{3/2} + C$$

$$= \frac{-2}{3} u\sqrt{u} + C$$

$$= \boxed{\frac{-2}{3} \cot x \cdot \sqrt{\cot x} + C}$$

May 19-10:22 AM

Evaluate $\displaystyle\int_0^{\sqrt{\pi}} x \cos x^2\, dx$

$u = x^2$

$du = 2x\, dx$

$\frac{du}{2} = x\, dx$

$x = 0 \rightarrow u = 0^2 = 0$

$x = \sqrt{\pi} \rightarrow u = (\sqrt{\pi})^2 = \pi$

$$\int_0^{\pi} \cos u \frac{du}{2}$$

$$= \frac{1}{2}\left[\sin u \Big|_0^{\pi} \right] = \frac{1}{2}[\sin \pi - \sin 0]$$

$$= \frac{1}{2}[0 - 0] = \boxed{0}$$

May 19-10:27 AM

Evaluate $\int_0^{\pi} \sec^2 \frac{x}{4}\, dx$

$u = \frac{x}{4}$

$du = \frac{1}{4}\, dx$

$4\,du = dx$

$x=0 \rightarrow u = \frac{0}{4} = 0$

$x=\pi \rightarrow u = \pi/4$

$$= \int_0^{\pi/4} \sec^2 u \; 4\,du = 4\int_0^{\pi/4} \sec^2 u\, du = 4 \tan u \Big|_0^{\pi/4}$$

$$= 4\left[\underbrace{\tan\frac{\pi}{4}}_{1} - \underbrace{\tan 0}_{0}\right]$$

$$= 4 \cdot 1 = \boxed{4}$$

May 19-10:32 AM

$\int_0^{\pi/2} \cos x \, \sin(\sin x)\, dx$

$u = \sin x$

$du = \cos x\, dx$

$x=0 \rightarrow u = \sin 0 = 0$

$x=\frac{\pi}{2} \rightarrow u = \sin\frac{\pi}{2} = 1$

$$= \int_0^{1} \sin u\, du = -\cos u \Big|_0^{1} = -\left[\cos 1 - \underbrace{\cos 0}_{1}\right]$$

$$= \boxed{1 - \cos 1}$$

May 19-10:39 AM

$$\int_0^1 x\sqrt{1-x}\,dx \qquad (0 \to u=1,\ 1 \to u=0)$$

$u=\sqrt{1-x}$

$u^2=1-x \to x=1-u^2$

$2u\,du=-dx$

$-2u\,du=dx$

$$=\int_1^0 (1-u^2)\cdot u\cdot -2u\,du$$

$$=-2\int_1^0 (1-u^2)u^2\,du$$

$$\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx$$

$$=-2\cdot -\int_0^1 (u^2-u^4)\,du = 2\left[\frac{u^3}{3}-\frac{u^5}{5}\right]\Big|_0^1$$

$$=2\left[\frac{1}{3}-\frac{1}{5}-0\right]$$

$$=2\cdot\frac{2}{15}=\boxed{\frac{4}{15}}$$

May 19-10:43 AM

$$\int \boxed{x}(x+5)^8\,dx$$

$u=x+5 \to u-5=x$

$du=dx$

$$=\int (u-5)u^8\,du=\int (u^9-5u^8)\,du$$

$$=\frac{u^{10}}{10}-\frac{5u^9}{9}+C$$

$$=\frac{1}{10}(x+5)^{10}-\frac{5}{9}(x+5)^9+C$$

May 19-10:51 AM

$$\int \frac{x^2}{\sqrt[3]{1+x^3}}\,dx$$

$u = 1 + x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$$= \int \frac{1}{\sqrt[3]{u}}\,\frac{du}{3} = \frac{1}{3}\int \frac{1}{u^{1/3}}\,du$$

$$= \frac{1}{3}\int u^{-1/3}\,du = \frac{1}{3}\cdot \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + C$$

$$= \frac{1}{3}\cdot\frac{3}{2}\sqrt[3]{u^2} + C$$

$$= \boxed{\frac{1}{2}\sqrt[3]{(1+x^3)^2} + C}$$

May 19-10:56 AM

$$\int \frac{x^2}{\sqrt[3]{1+x^3}}\,dx$$

$u = \sqrt[3]{1+x^3}$

$u^3 = 1 + x^3$

$\cancel{3}u^2\,du = \cancel{3}x^2\,dx$

$u^2\,du = x^2\,dx$

$$= \int \frac{u^2\,du}{u}$$

$$= \int u\,du = \frac{u^2}{2} + C$$

$$= \frac{1}{2}\left(\sqrt[3]{1+x^3}\right)^2 + C$$

$$= \frac{1}{2}\sqrt[3]{(1+x^3)^2} + C$$

Find f_{ave} for

1) $f(x) = \frac{x}{\sqrt{3+x^2}}$ on $[1,3]$

2) $f(x) = \cos^4 x \sin x$ on $[0,\pi]$

Make sure you have ready by wednesday morning to turn in.

May 19-11:03 AM

Class QZ 23

Find f_{ave} for $f(x) = \sqrt[3]{x}$ on $[1, 8]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{ave} = \frac{1}{8-1}\int_1^8 \sqrt[3]{x}\,dx$$

$$= \frac{1}{7}\int_1^8 x^{1/3}\,dx$$

$$= \frac{1}{7}\cdot \frac{x^{4/3}}{4/3}\Big|_1^8$$

$$= \frac{3}{28}\left[8^{4/3} - 1^{4/3}\right]$$

$$= \frac{3}{28}\left[\left(2^3\right)^{4/3} - 1\right] = \frac{3}{28}\left[16-1\right]$$

$$= \frac{3}{28}\cdot 15$$

$$= \boxed{\frac{45}{28}}$$

May 19-11:09 AM